Calculation of Ball Trajectories With and Without The Effect of Air Resistance

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Equations of Motion for A Trajectory Without Air Resistance

- The flight of a projectile with initial velocity v_o initial angle θ (with respect to horizontal) is described by the following sets of equations (neglecting air resistance):

- Horizontal Motion | Vertical Motion

$$a_x = 0$$

$$v_x = v_{o,x}$$

$$x = v_{o,x} t$$

$$a_y = -g$$

$$v_y = v_{o,y} - gt$$

$$y = v_{o,y} t - \tfrac{1}{2} g t^2$$

Where

$v_{o,x} = v_o \cos\theta$, $v_{o,y} = v_o \sin\theta$ and the acceleration of gravity g = 9.81 m/s^2

- Thus, there is an analytical solution for the position as a function of time

Equations of Motion for A Trajectory Including Air Resistance

- The flight of a projectile with initial velocity v_o initial angle θ (with respect to horizontal) is described by the following sets of equations (including air resistance):

- Horizontal Motion

$$\frac{dv_x}{dt} = \frac{f_{drag,x}(t, v_x)}{m}$$

$$\frac{dx}{dt} = v_x(t)$$

Vertical Motion

$$\frac{dv_y}{dt} = a_y = \frac{f_{drag,y}(t, v_y)}{m}$$

$$\frac{dy}{dt} = v_y(t)$$

Where the force of drag

$$f_{drag}(t) = -\tfrac{1}{2} C_d \rho A v^2$$

$$\textit{where } C_d = \textit{numerical drag coefficient} \approx 0.5 \textit{ for a sphere}$$

$$\rho = \textit{density of air} = 1.225\, kg/m^3 \textit{ at sea level}$$

$$A = \textit{cross-sectional area} = \tfrac{1}{4}\pi d^2 \textit{ for a sphere}$$

$$v = \textit{velocity}$$

A numerical solution of the differential equations must be used to get the position information (I used a C++ program to solve them using a 4th order Runga-Kutta method)

Minimum and Maximum Ranges To Fire The Ball At Goal

v_o = 12 m/s, ball diameter = 7", mass of ball = 185 g

Assuming that we would like to used a fixed launch angle to minimize the complications of making adjustments on the fly in the competition, we set the fixed angle such that the peak height of the trajectory allows the ball to pass just below the top of the goal. We can then calculate the minimum and maximum range from which to shoot the ball and still clear the goal. Here are these ranges with and without the effects of air resistance for various heights for the launcher.

	WITHOUT AIR RESISTANCE			***WITH AIR RESISTANCE***		
Height of Launcher (inches)	**Min Range ft**	**Max Range ft**	**Optimal Angle degrees**	**Min Range ft**	**Max Range ft**	**Optimal Angle degrees**
60	6.4	31.0	25.5	5.8	25.1	27.7
55	7.2	31.5	26.7	6.4	25.4	29.2
50	7.9	32.0	28.0	6.8	26.0	30.6
45	8.6	32.4	29.1	7.4	25.8	32.0

Calculation of The Range That Ball Will Travel With and Without Air Resistance When $\theta = 30^\circ$

v_o = 12 m/s, ball diameter = 7", mass of ball = 185 g

	With Drag	***W/O Drag***
Height of Launcher (inches)	**Range ft**	**Range ft**
60	36.3	49.1
55	35.9	48.4
50	35.6	48.0
45	35.2	47.4
40	34.8	46.8

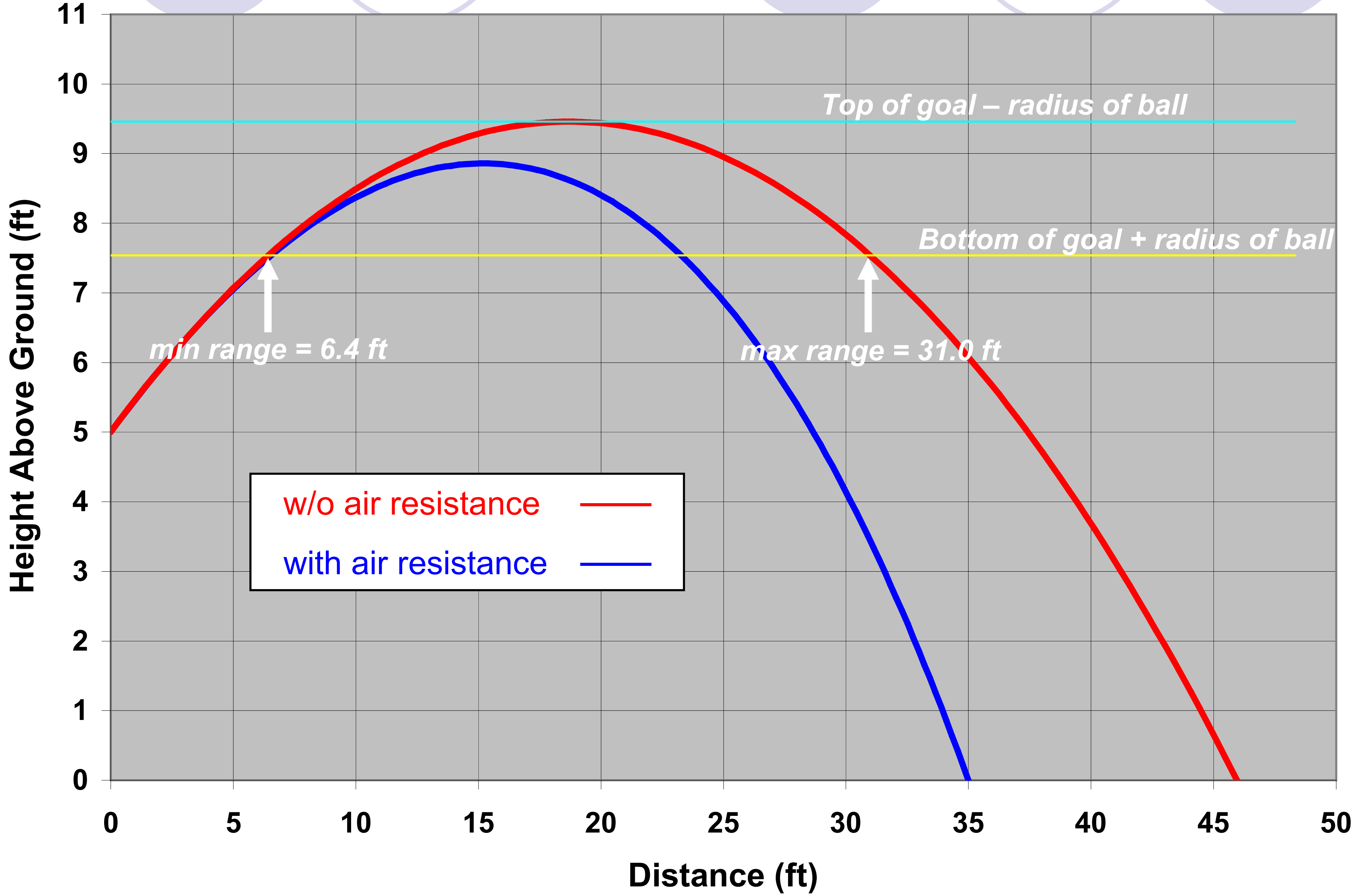
From section 4.3.1 of the 2006 FIRST Robotics Competition Documents:

<S02> Muzzle Velocity - No ROBOT may throw a ball with an exit velocity of greater than 12 m/s (26.8 mph). As a reference, a ball traveling at this velocity when leaving the ROBOT at an *angle of 30º* from horizontal with no spin *will travel approximately 35 feet*. A robot that violates this rule will be considered unsafe per <S01>.

Thus, my numerical simulation that included air drag appears to be in agreement with the expectations from FIRST

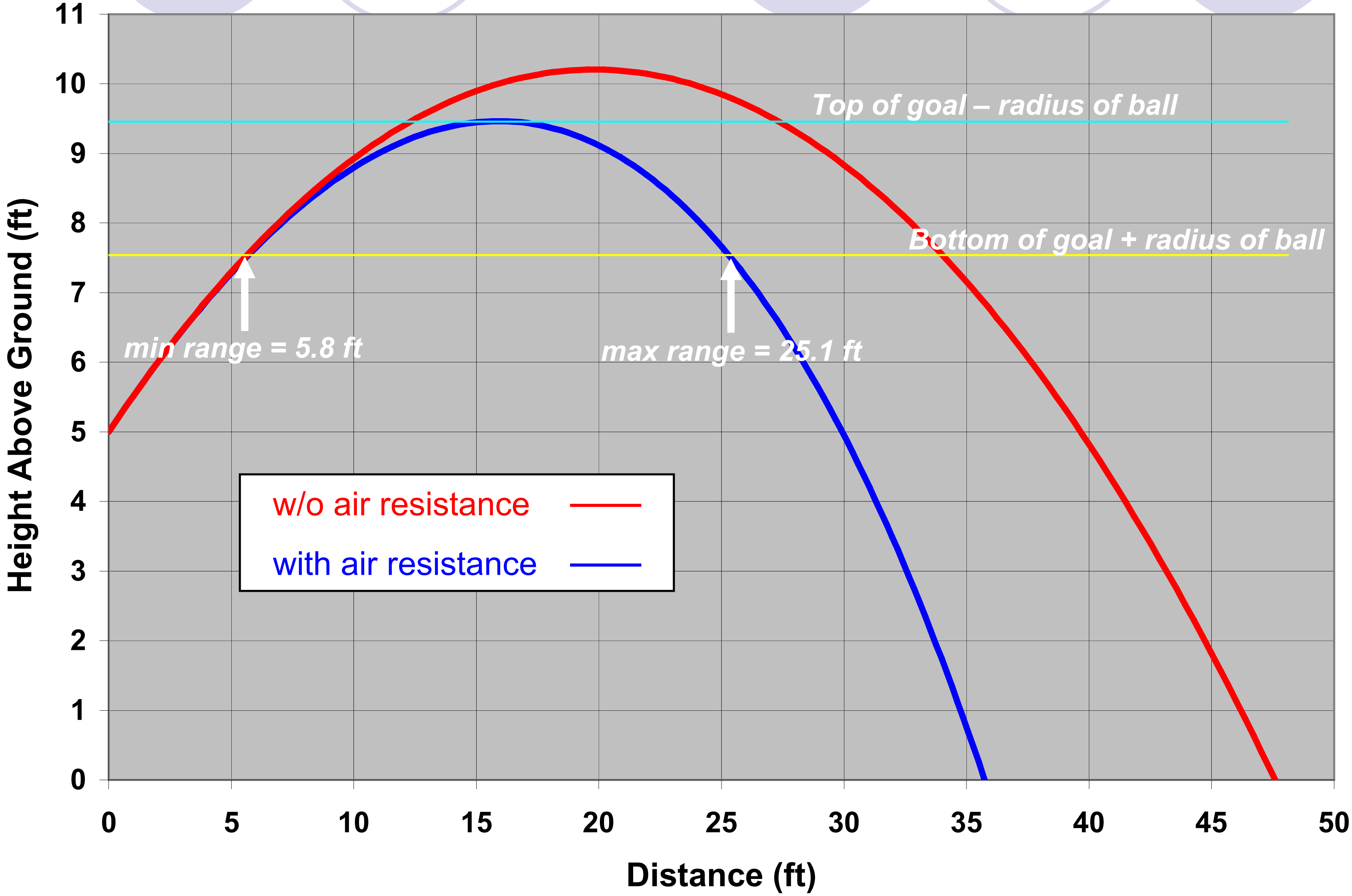
Optimal Trajectory Without Air Resistance

v_o = 12 m/s, θ = 25.5 degrees, height of launcher = 60”

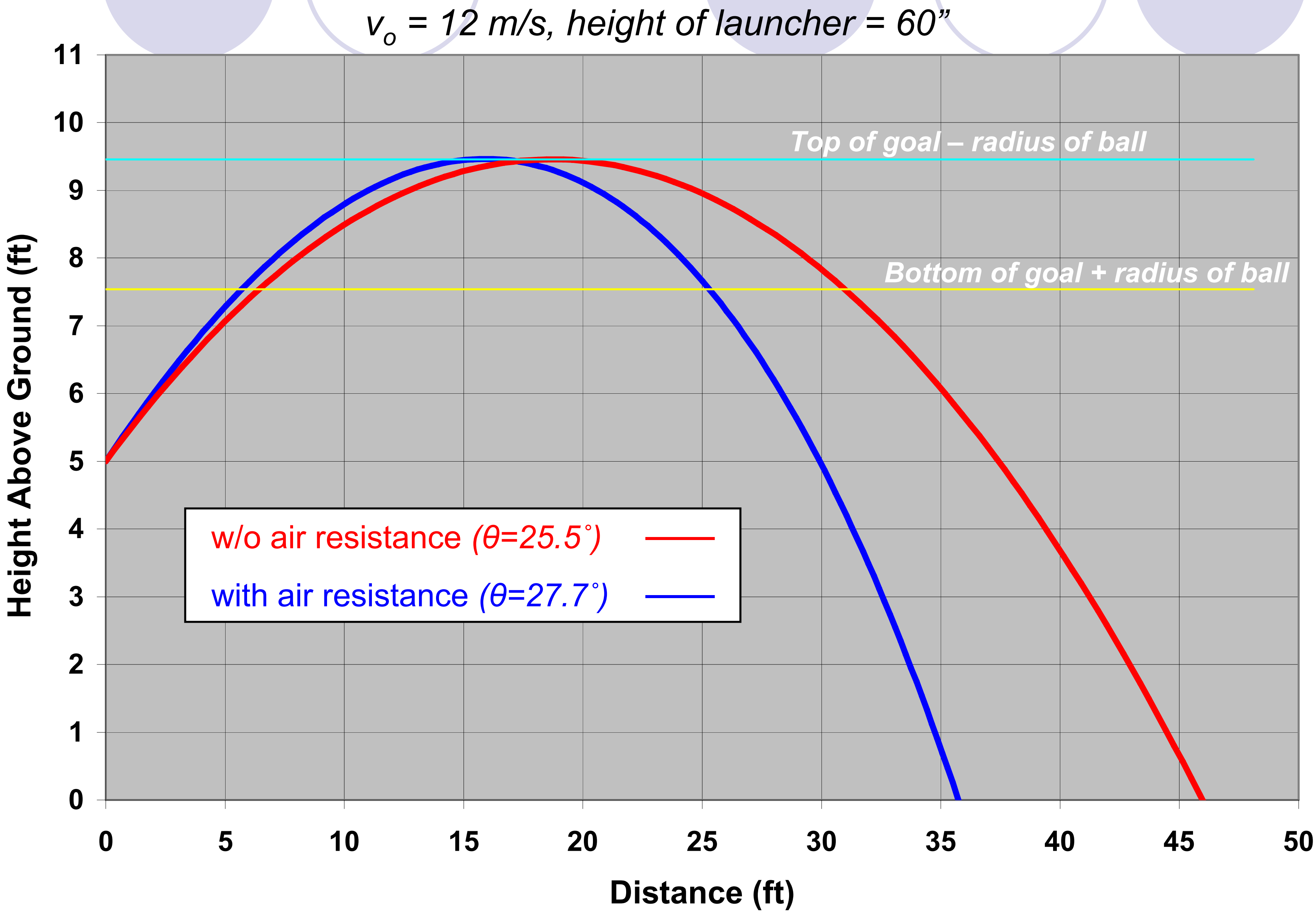


Optimal Trajectory With Air Resistance

v_o = 12 m/s, θ = 27.7 degrees, height of launcher = 60"



Comparison of Optimal Trajectories With and Without Air Resistance



References

- Trajectory equations ignoring air resistance: http://hyperphysics.phy-astr.gsu.edu/hbase/traj.html
- Equation for the force of air drag: http://hyperphysics.phy-astr.gsu.edu/hbase/hframe.html
- Discussion regarding the effect of spin (Magnus effect): http://carini.physics.indiana.edu/E105/spinning-balls.html
- Discussion about how to calculate the trajectory including air drag: http://www.pdas.com/bb1.htm